Support Vector Machines

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Constraint Violation

At times, having a hard constraint may not be so useful especially when there are outliers or noise in the dataset. In such a case we allow for the constraints to be minimally violated. An example is show below:



To introduce this effect, we add slack variables to our model which allow constraint violation. The slack variable $\xi_i \in [0, 1)$ enables the point x_i to be in between the margin and on the correct side of the hyperplane. This is called **margin violation**. If $\xi_i > 1$, then the point is misclassified.

The corresponding objective function of the SVM becomes:

$$\min_{\mathbf{w}\in\mathbb{R}^d,\xi\in\mathbb{R}^+}\|\mathbf{w}\|^2 + C\sum_i^N \xi_i$$

subject to:

$$y_i\{w^T x + b\} \ge 1 - xi_i \; ; \; \forall \; i = 1...N$$

KKT Conditions

- Stationarity: Calculate the lagrangian and set its differential wrt each variable to 0.
- Complementary Slackness: Multiplication of multiplier and the h(x) condition is 0.
- Primal Feasibility: $h(x) \leq 0, l(x) = 0.$
- Dual Feasability: Multipliers are greater than 0.



Figure 9: Introduction of slack variables

Figure 1: Introduction of Slack Variables